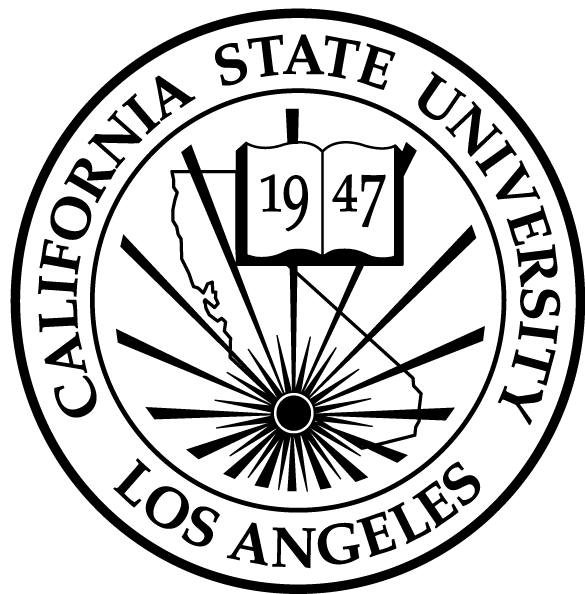
**California State University, Los Angeles**



ME 409 – Mechanical Engineering Analysis

Prepared by:

William Santiago (301579504)

David Arenas (300089950)

Edward Gonzalez (300593922)

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Professor: Usama Tohid, P.E.

**1. Abstract**

*The purpose of this study is to analyze a non-homogenous system. To better understand this, the approach for calculating our results will be by building a mathematical model with a programming aid such as Matlab. This mathematical model is used towards the non-homogenous system which in this case is known as a mixture problem with two separate tanks. The method used in this study for calculating is known as the Method of Undetermined Coefficients. The findings reveal different equations or solutions when using the method of Undetermined Coefficients when changing the values of Tank 1 and Tank 2. Furthermore, a total of 50 combinations were made to fully understand the methods used and determining the amounts of salt for both Tank 1 and Tank2.*

**Nomenclatures:**

|  |  |  |
| --- | --- | --- |
| Greek Symbol | Meaning | Unit |
| *λ* | Scalar (Real or Complex Number) |  |

**2. Introduction**

One of the main applications of differential equations in general is known as Modeling. Modeling is the process of writing a differential equation to describe a physical situation that is occurring. Most of the differential equations that are used in industry for engineers are there because somebody, at some point modeled a situation and came up with the differential equation that we are using. There are times when the modeling problem is a non-homogenous system, enabling the undetermined coefficients method to be used when calculating the differential equation.

**2.1 Overview and Purpose**

In the modeling system problem that we will be solving starts with a substance that is being dissolved in a liquid. This liquid entering the tank may or may not contain more of the substance that is being dissolved in it. The liquid leaving the tank will contain the substance dissolved in it. We know that if *Q(t)* gives the amount of the substance that is being dissolved in the liquid in the tank at any time *t* we want to develop a differential equation that when it is solved, it will give us an expression for *Q(t)*.

The main assumption that we are making in this problem is that the concentration of the substance in the liquid is uniform throughout the tank. Taking into consideration that there are two tanks that are being interconnected with liquid while potentially entering both and with an exit.

Our team has made the following assumptions for this situation involving two tanks. First of all, the inflow and outflow from each tank are equal, or in a simpler detail the volume in each tank is constant. Next, the concentration of contaminate in each tank will be the same at each point in the tank. Also, the concentration of contaminate in the outflow of the tank 1 will be the same as the concentration in tank 1, while the concentration of contaminate in the outflow from tank 2 will be the same as the concentration in tank 2. In addition, the outflow from tank 1 will be exiting tank 1 and reaching tank 2. The total liquid will be exiting the system completely.

As mentioned before, our team is dealing with a system containing two tanks. The first tank initially contains 200 gallons of water with 160 lb of salt within being dissolved. The second tank initially contains 100 gallons of pure water while the liquid is pumped throughout the system and the mixtures are kept uniform while being stirred. Our team proposed the idea for calculating the solution with the method of undetermined coefficients for this non-homogenous system.

**2.2 Statement of the Problem**

1.) Tank *T*1 in Fig. 101 initially contains

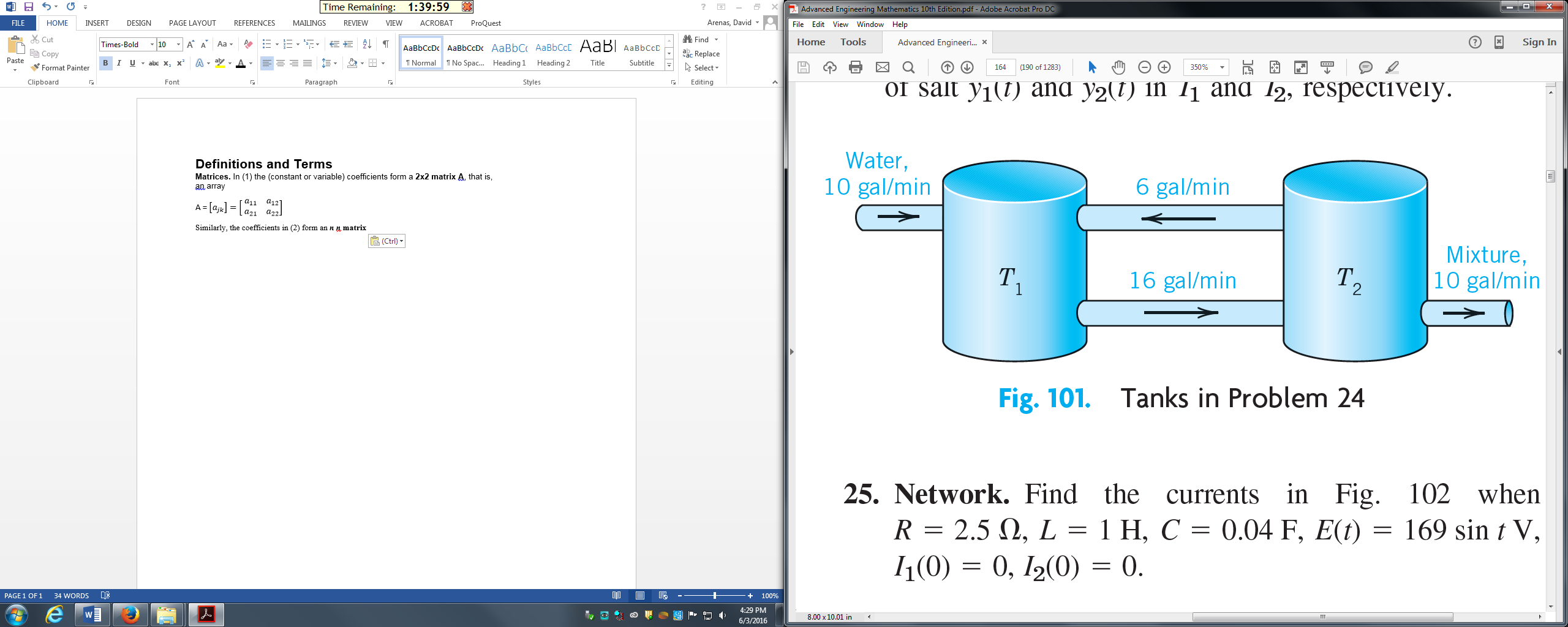
200 gal of water in which 160 lb of salt are dissolved.

Tank *T*2 initially contains 100 gal of pure water. Liquid

is pumped through the system as indicated, and the

mixtures are kept uniform by stirring. Find the amounts

of salt *y*1(*t*) and *y*2(*t*) in *T*1 and *T*2, respectively.



2.) Repeat the same problem for which T1 contains: 50, 100, 150, 200, 250 gal of water and T2 contains 10, 20, 50, 200, 500 gal of water. (25 combinations)

3.) Repeat the same problem if the flow from tank T1 to T2 changes from 1,2,5,10,20 gal/min and T2 to T1 1,2,5,10,100 gal/min. (25 combinations)

**3. Theory and Principles**

In this study, most of our linear systems will consist of two linear Ordinary Differential Equations (ODEs) in two unknown functions

In addition, a linear system of first-order Ordinary Differential Equations (ODEs) in unknown functions is of the form

**3.1 Matrices**

The (constant or variable) coefficients form a **2x2 matrix** , that are,

an array

In addition, the coefficients in the equation form an ***n* matrix**

The are called **entries**, the horizontal lines **rows**, and the vertical lines **columns**. Thus, in the equation, the first row is, the second row is and the first and second columns are

and .

In the “***double subscript notation***” for various entries, the first subscript is that one that denotes the *row* and the second the *column* in which the entry stands. Similarly in the previous equation. The **main diagonal** is the diagonal , hence .

We shall need only **square matrices**, that is, matrices with the same number of rows and columns, as in the previous equations in this project.

**3.2 Vectors**

A **column vector** with **components** such as theseis of the form

Similarly, a **row vector v** is of the following form below,

**3.3 Equality**

Now we move on to calculations with matrices and vectors. Two matrices will be *equal* if and only if corresponding entries are equal as well. For example, let’s take this for instance, Thus for , let

Then if and only if

Now, two column vectors (or two row vectors) are *equal* if and only if they both have components and corresponding components are equal as well. For example, let

**3.4 Addition**

This is performed by adding the corresponding entries (or components). The matrices must both be, and vectors must both have the same number of components. For example, for ,

**3.5 Matrix Multiplication.**

The product **(**in this order) of two matrices and  **is the matrix**  with entriesthe following;

Let us keep caution with the following note; Matrix multiplication is **not commutative.**

Furthermore, a multiplication of an matrix **A** by a vector **x** with components is defined by the same rule: is the vector with the components

**3.6 Differentiation**

Now we will move on to Systems of ODEs as Vector Equations. The *derivative* of a matrix (or vector) with variable entries (or components) is obtained by differentiating each entry (or component). By using the matrix multiplication and differentiation, we can now write the following;

Similarly, the matrix **A** and a column vector with components, namely, The vector equation is equivalent to two equations for the components, and these are precisely the two ODEs in the first equation.

**3.7 Inverse Matrix**

The unit matrix I is the matrix with main diagonal 1, 1,…, 1 and all of the other entries that are zero. If, for a given matrix A, there is a matrix B such that AB = BA = I, then A is called nonsingular and B is called the Inverse of A and is denoted by so the following is true;

The inverse will exist if the determinant det A of A is not zero. If A has not inverse, then we will call it a singular. For example is n = 2, then

Where the determinant of A is

det A = =

**3.8 Eigenvalues, Eigenvectors**

This next process is quite important and useful. The Eigenvalues and eigenvectors will be very important during the process of calculating our solution.

Let be an matrix. Consider the following equation;

where λ is a scalar (a real or complex number) to be determined and **x** is a vector to be determined. Now, for every λ, a solution is . A scalar λ such that it holds for some vector is called an **eigenvalue** of **A**, and this vector is called an **eigenvector** of **A** corresponding to this eigenvalue λ.

These are linear algebraic equations in the unknowns (the components of **x**). For these equations to have a solution , the determinant of the coefficient matrix must be zero. For example, we only need this only for . Then the following is

In components,

Now, is singular if and only if its determinant , called the **characteristic determinant** of **A** (also for general *n*), is zero. This will give us the following;

This quadratic equation in λ is called the **characteristic equation** of **A**. Its solutions are the eigenvalues of **A**. First determine these. Then use with to determine an eigenvector of **A** corresponding to . Finally use with to find an eigenvector of **A** corresponding to . Note that if **x** is an eigenvector of **A**, so is with any

**3.9 Nonhomogeneous Linear Systems of ODEs**

We will now move on to the situation where the linear system of the Ordinary Differential Equation is Non-homogenous. Here we will use the method for solving nonhomogeneous linear systems of ODEs;

where the vector is not identically zero. We assume and the entries of the matrix to be continuous on some interval of the *-*axis. From a general solution of the homogeneous system on and a **particular solution** of **.**

is called a **general solution** of (1) on because it includes every solution of on .

**3.10 Method of Undetermined Coefficients**

Now for the actual use of the method. Just as for a single ODE, this method is suitable only if the entries of **A** are constants and the components of **g** are constants, positive integer powers of, exponential functions, or cosines and sines. For instance, in such a case, a particular solution is assumed in a form similar to **g**; for instance, if **g** has components quadratic in, with **u**, **v**, **w** to be determined by substitution into **.**

**4. Data Analysis**

**4.1. Analysis of Calculation and Tables**

**4.1.1 Sample Calculation using Method of Undetermined Coefficients for Original Problem**

The following calculation demonstrates a sample calculation or the original problem in finding the equations to determine the amount of salt in Tank 1 and Tank 2. This was done by hand before using the aid of Matlab to see if the solution would match. Verification was very crucial for our team. Here is our sample calculation:

**Given:**

, , Inflow

, , Inflow

**Required:**

**Solution:**

*Free Body Diagram (FBD)*:

In

Out

*Free Body Diagram (FBD)*:

Out

Out

In

Eigenvalues

Eigenvectors for

Eigenvectors for

**4.1.2 Sample Table Data for Part 1**

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| **Eigenvalues** | **Vector V11** | **Vector V22** | **Constants for y1** | **Constants for y2** |
| -0.04 | 1 | 1 | 40 | -80 |
| -0.2 | -2 | 0.666666667 | 120 | 80 |
|  |  |  |  |  |
| T(min) | y1 |  | T(min) | y2 |
| 0 | 0 |  | 0 | 160 |
| 1 | 11.36469489 |  | 1 | 148.0439628 |
| 2 | 20.22370403 |  | 2 | 137.5867634 |
| 3 | 27.04870405 |  | 3 | 128.3829178 |
| 4 | 32.22518599 |  | 4 | 120.2304132 |
| 5 | 36.06810495 |  | 5 | 112.962868 |
| 6 | 38.83469193 |  | 6 | 106.4431118 |
| 7 | 40.7349422 |  | 7 | 100.5579275 |
| 8 | 41.94020153 |  | 8 | 95.21374517 |
| 9 | 42.59019503 |  | 9 | 90.33311466 |
| 10 | 42.79878102 |  | 10 | 85.85181685 |
| 11 | 42.65866102 |  | 11 | 81.71649686 |
| 12 | 42.24523508 |  | 12 | 77.88272515 |
| 13 | 41.61975758 |  | 13 | 74.31340888 |
| 14 | 40.8319201 |  | 14 | 70.97749017 |
| 15 | 39.92196542 |  | 15 | 67.84887907 |
| 16 | 38.92241761 |  | 16 | 64.90557904 |
| 17 | 37.85949779 |  | 17 | 62.12896988 |
| 18 | 36.75428268 |  | 18 | 59.50321961 |
| 19 | 35.62365241 |  | 19 | 57.01480212 |
| 20 | 34.48106602 |  | 20 | 54.65210125 |
| 21 | 33.33719573 |  | 21 | 52.40508588 |
| 22 | 32.20044574 |  | 22 | 50.265043 |
| 23 | 31.07737643 |  | 23 | 48.22435836 |
| 24 | 29.97305111 |  | 24 | 46.2763362 |
| 25 | 28.89131953 |  | 25 | 44.41505082 |
| 26 | 27.8350494 |  | 26 | 42.63522441 |
| 27 | 26.80631558 |  | 27 | 40.93212632 |
| 28 | 25.80655447 |  | 28 | 39.3014899 |
| 29 | 24.83669009 |  | 29 | 37.7394439 |
| 30 | 23.89723678 |  | 30 | 36.24245552 |
| 31 | 22.98838298 |  | 31 | 34.80728338 |
| 32 | 22.11005945 |  | 32 | 33.43093835 |
| 33 | 21.26199471 |  | 33 | 32.11065096 |
| 34 | 20.44376014 |  | 34 | 30.84384424 |
| 35 | 19.65480656 |  | 35 | 29.62811095 |
| 36 | 18.89449383 |  | 36 | 28.46119447 |
| 37 | 18.16211485 |  | 37 | 27.34097272 |
| 38 | 17.45691484 |  | 38 | 26.26544449 |
| 39 | 16.7781069 |  | 39 | 25.23271794 |
| 40 | 16.12488443 |  | 40 | 24.24100066 |
| 41 | 15.4964311 |  | 41 | 23.28859122 |
| 42 | 14.8919287 |  | 42 | 22.37387182 |
| 43 | 14.31056337 |  | 43 | 21.49530198 |
| 44 | 13.75153046 |  | 44 | 20.65141298 |
| 45 | 13.21403827 |  | 45 | 19.84080298 |
| 46 | 12.69731094 |  | 46 | 19.06213271 |
| 47 | 12.20059054 |  | 47 | 18.31412165 |
| 48 | 11.72313867 |  | 48 | 17.59554461 |
| 49 | 11.26423755 |  | 49 | 16.90522857 |
| 50 | 10.82319066 |  | 50 | 16.24204999 |
| 51 | 10.39932324 |  | 51 | 15.60493212 |
| 52 | 9.991982377 |  | 52 | 14.99284276 |
| 53 | 9.600537 |  | 53 | 14.40479206 |
| 54 | 9.224377723 |  | 54 | 13.8398305 |
| 55 | 8.862916533 |  | 55 | 13.29704707 |
| 56 | 8.515586415 |  | 56 | 12.77556749 |
| 57 | 8.181840898 |  | 57 | 12.27455263 |
| 58 | 7.861153561 |  | 58 | 11.79319692 |
| 59 | 7.553017491 |  | 59 | 11.33072697 |
| 60 | 7.256944726 |  | 60 | 10.88640016 |
| 61 | 6.972465681 |  | 61 | 10.45950339 |
| 62 | 6.69912856 |  | 62 | 10.04935181 |
| 63 | 6.436498779 |  | 63 | 9.655287691 |
| 64 | 6.184158374 |  | 64 | 9.276679284 |
| 65 | 5.941705431 |  | 65 | 8.912919799 |
| 66 | 5.708753516 |  | 66 | 8.563426371 |
| 67 | 5.484931121 |  | 67 | 8.227639104 |
| 68 | 5.269881115 |  | 68 | 7.905020151 |
| 69 | 5.063260218 |  | 69 | 7.595052828 |
| 70 | 4.864738488 |  | 70 | 7.297240776 |
| 71 | 4.673998813 |  | 71 | 7.011107148 |
| 72 | 4.490736436 |  | 72 | 6.736193836 |
| 73 | 4.314658476 |  | 73 | 6.47206073 |
| 74 | 4.145483483 |  | 74 | 6.218285006 |
| 75 | 3.982940997 |  | 75 | 5.97446044 |
| 76 | 3.826771123 |  | 76 | 5.740196757 |
| 77 | 3.676724128 |  | 77 | 5.515119 |
| 78 | 3.532560043 |  | 78 | 5.298866926 |
| 79 | 3.39404829 |  | 79 | 5.091094428 |
| 80 | 3.260967315 |  | 80 | 4.891468979 |
| 81 | 3.133104237 |  | 81 | 4.699671097 |
| 82 | 3.01025451 |  | 82 | 4.515393834 |
| 83 | 2.892221599 |  | 83 | 4.338342281 |
| 84 | 2.77881667 |  | 84 | 4.168233096 |
| 85 | 2.669858285 |  | 85 | 4.004794051 |
| 86 | 2.565172115 |  | 86 | 3.847763595 |
| 87 | 2.464590663 |  | 87 | 3.696890434 |
| 88 | 2.367952996 |  | 88 | 3.551933129 |
| 89 | 2.275104489 |  | 89 | 3.41265971 |
| 90 | 2.185896577 |  | 90 | 3.278847303 |
| 91 | 2.10018652 |  | 91 | 3.150281775 |
| 92 | 2.01783717 |  | 92 | 3.026757389 |
| 93 | 1.938716759 |  | 93 | 2.908076476 |
| 94 | 1.862698683 |  | 94 | 2.794049119 |
| 95 | 1.7896613 |  | 95 | 2.684492847 |
| 96 | 1.719487741 |  | 96 | 2.579232345 |
| 97 | 1.652065714 |  | 97 | 2.478099172 |
| 98 | 1.587287334 |  | 98 | 2.380931492 |
| 99 | 1.525048942 |  | 99 | 2.287573816 |
| 100 | 1.465250946 |  | 100 | 2.197876749 |

**4.2 Data Analysis Plot Graphs**

The following analysis was made with the aid of Matlab programming. The following graphs demonstrate the mathematical model for the differential equation. The differential equation was then plotted to demonstrate the function over time. The team began with graphing a plot for the Part 1. Following up with the modeling system, the graphs were then made for Part 2 and Part 3 to demonstrate the change of gallons of water and change of flow of the water, respectively. The following graphs show the complete run for Part A since it was only 1 run. Next, the graphs for Part 2 and Part 3 only show for the first 5 runs, respectively. This was done to save time and demonstrate how the function will be changing over time.

**4.2.1 Graph for Part 1**

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**Graph 1: Part 1 1st Run**

**4.2.2 Graphs for Part 2**

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**Graph 2: Part 2 1st Run**

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**Graph 3: Part 2 2nd Run**

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**Graph 4: Part 2 3rd Run**

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**Graph 5: Part 2 4th Run**

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**Graph 6: Part 2 5th Run**

**4.2.3 Graphs for Part 3**

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**Graph 7: Part 3 1st Run**

****

**Graph 8: Part 3 2nd Run**

****

**Graph 9: Part 3 3rd Run**

****

**Graph 10: Part 3 4th Run**

****

**Graph 11: Part 3 5th Run**

**5. Results and Discussion**

Our team managed to compute a differential equation model for the non-homogeneous system involving two tanks. The mixture problem was an ideal situation for our understanding of modeling systems. The eigenvectors and eigenvalues were shown to be quite different when solving each mathematical model. For Part 1, this was basic due to the fact that we had everything given in terms of what we needed to solve the non-homogenous system. For Part 2, we had to change the gallons for the tank1 and tank 2. When this was applied, the values were prompted to change. This concept was also seen in Part 3, when the flow of water was changed for tank 1 and tank 2. When the flow was changed as well, the values given to us were quite different as well. Each graph shows a differential equation for tank 1 and tank 2. The graphs show the differences between times for each tank and also it displays differences of each. At some point, both differential equations will intersect at a neutral point in which shows the mixture of liquid and a substance (salt) combine into one and thus revealing a mixture flow throughout the function. Overall, the graphs give us an understanding where the mixture is occurring and conclude this understanding of non-homogenous system. The results give us a great ideal of information to see how this non-homogenous system behaves as a mathematical model in a differential equation.

**6. Conclusion**

This cumulative study on modeling a differential equation for a non-homogeneous system involving a mixture of two tanks was a great opportunity to broaden our understanding. This situation gave us a broad idea of what modeling differential equation is like and how to show a functional mathematical model with the aid of programming such as Matlab. The values show different outcomes when the differential equation for the modeling system were created. This project allowed us to see how the differential equations behave when plotted and helps us view what is occurring over time. The overall task was completed for the first part with a great understanding. The team then proceeded to repeat the same problem when T1 contained different values such as 50, 100, 150, 200, 250 gal of water and while T2 contained 10, 20, 50, 200, 500 gal of water. This allowed us to see different situations for our modeling systems. Once this was completed, the team repeated the same problem when the flow from tank T1 to T2 changes with different values such as 1,2,5,10,20 gal/min and while T2 to T1 with the following values 1, 2,5,10,100 gal/min. We did a total of 25 combinations for both the second and third parts of this project giving us a total of 50 combinations. Overall, the study of modeling a non-homogenous system was enlightening and can be concurrently applied practically in research and design within the engineering industry.

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**8. Appendix**

**8.1 Part 1 Matlab Code**

**8.1.1 Run 1**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=100;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=100;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 40 e^(-0.04)t + 120 e^(-0.2)t

% Y2 = 26.68 e^(-0.04)t - 240 e^(-0.2)t

**8.2 Part 2 Matlab Code**

**8.2.1 Run 1**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=10;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=10;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 13.98 e^(-0.1844)t + 146.01 e^(-1.7356)t

% Y2 = 3.1615 e^(-0.1844)t - 344.49 e^(-1.7356)t

**8.2.2 Run 2**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 31.01 e^(-0.1681)t + 128.98 e^(-0.9519)t

% Y2 = 15.70 e^(-0.1681)t – 271.70 e^(-0.9519)t

**8.2.3 Run 3**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=50;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=50;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 80 e^(-0.124)t + 80 e^(-0.516)t

% Y2 = 130.4 e^(-0.124)t - 130.4 e^(-0.516)t

**8.2.4 Run 4**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=200;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=200;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 18.0323 e^(-0.3549)t + 141.9677 e^(-0.0451)t

% Y2 = -20.98 e^(-0.3549)t + 1300.99 e^(-0.0451)t

**8.2.5 Run 5**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=500;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=500;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 6.5153 e^(-0.3328)t + 153.487 e^(-0.0192)t

% Y2 = -6.9316 e^(-0.3328)t + 3846.92 e^(-0.0192)t

**8.2.6 Run 6**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=10;

a12=b/c

d=16;

f=100;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=100;

a21=d/f

g=10;

h=10;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 6.5153 e^(-0.0962)t + 153.4847 e^(-1.6638)t

% Y2 = 0.6932 e^(-0.0962)t - 384.69 e^(-1.6638)t

**8.2.7 Run 7**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=100;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=100;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 13.98 e^(-0.0922)t + 146.01 e^(-0.8678)t

% Y2 = 3.1579 e^(-0.0922)t - 344.49 e^(-0.8678)t

**8.2.8 Run 8**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=50;

a12=b/c

d=16;

f=100;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=100;

a21=d/f

g=10;

h=50;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 40 e^(-0.08)t + 120 e^(-0.40)t

% Y2 = 26.68 e^(-0.08)t - 240 e^(-0.40)t

**8.2.9 Run 9**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=200;

a12=b/c

d=16;

f=100;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=100;

a21=d/f

g=10;

h=200;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 40 e^(-0.2)t + 120 e^(-0.04)t

% Y2 = -53.32 e^(-0.2)t + 480 e^(-0.04)t

**8.2.10 Run 10**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=500;

a12=b/c

d=16;

f=100;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=100;

a21=d/f

g=10;

h=500;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 13.98 e^(-0.1736)t + 120 e^(-0.0184)t

% Y2 = -15.80 e^(-0.1736)t + 1722.46 e^(-0.0184)t

**8.2.11 Run 11**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=10;

a12=b/c

d=16;

f=150;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=150;

a21=d/f

g=10;

h=10;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 4.23 e^(-0.065)t + 153.76 e^(-1.64)t

% Y2 = 0.294 e^(-0.065)t - 393.40 e^(-1.64)t

**8.2.12 Run 12**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=150;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=150;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 8.9071 e^(-0.0632)t + 151.0929 e^(-0.8434)t

% Y2 = 1.291 e^(-0.0632)t - 371.0690 e^(-0.8434)t

**8.2.13 Run 13**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=50;

a12=b/c

d=16;

f=150;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=150;

a21=d/f

g=10;

h=50;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 25.1205 e^(-0.0578)t + 134.8795 e^(-0.3688)t

% Y2 = 10.221 e^(-0.0578)t – 294.67 e^(-0.3688)t

**8.2.14 Run 14**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=200;

a12=b/c

d=16;

f=150;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=150;

a21=d/f

g=10;

h=200;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 61.6467 e^(-0.1515)t + 98.3533 e^(-0.0352)t

% Y2 = -92.026 e^(-0.1515)t + 234.248 e^(-0.0352)t

**8.2.15 Run 15**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=500;

a12=b/c

d=16;

f=150;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=150;

a21=d/f

g=10;

h=500;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 22.2404 e^(-0.1210)t + 137.7596 e^(-0.0176)t

% Y2 = -26.641 e^(-0.1210)t + 1022.2 e^(-0.0176)t

**8.2.16 Run 16**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=10;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=10;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 3.1303 e^(-0.0491)t + 156.8693 e^(-1.6309)t

% Y2 = 0.1615 e^(-0.0491)t – 405.4924 e^(-1.6309)t

**8.2.17 Run 17**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 6.1557 e^(-0.0481)t + 153.4847 e^(-0.8319)t

% Y2 = 0.6932 e^(-0.0481)t - 384.69 e^(-0.8319)t

**8.2.18 Run 18**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=50;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=50;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 18.0323 e^(-0.0451)t + 141.9677 e^(-0.3549)t

% Y2 = 5.2473 e^(-0.-0451)t - 325.24 e^(-0.3549)t

**8.2.19 Run 19**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=200;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=200;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 80 e^(-0.0310)t + 80 e^(-0.1290)t

% Y2 = 130.64 e^(-0.0310)t - 130.64 e^(-0.1290)t

**8.2.20 Run 20**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=500;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=500;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 31.0102 e^(-0.0952)t + 128.98 e^(-0.0168)t

% Y2 = -39.258 e^(-0.0952)t + 679.26 e^(-0.0168)t

**8.2.21 Run 21**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=10;

a12=b/c

d=16;

f=250;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=250;

a21=d/f

g=10;

h=10;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 2.4835 e^(-0.0394)t + 157.5156 e^(-1.6246)t

% Y2 = 0.1018 e^(-0.0394)t – 409.7 e^(-1.6246)t

**8.2.22 Run 22**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=250;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=250;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 5.1315 e^(-0.0388)t + 154.8586 e^(-0.8252)t

% Y2 = 0.4315 e^(-0.0388)t – 392.96 e^(-0.8252)t

**8.2.23 Run 23**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=50;

a12=b/c

d=16;

f=250;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=250;

a21=d/f

g=10;

h=50;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 13.98 e^(-0.0369)t + 146.0110 e^(-0.3471)t

% Y2 = 3.1594 e^(-0.0369)t + 344.498 e^(-0.3471)t

**8.2.24 Run 24**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=200;

a12=b/c

d=16;

f=250;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=250;

a21=d/f

g=10;

h=200;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 65.6316 e^(-0.0275)t + 94.3684 e^(-0.1165)t

% Y2 = 79.9458 e^(-0.0275)t + 165.276 e^(-0.1165)t

**8.2.25 Run 25**% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=500;

a12=b/c

d=16;

f=250;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=250;

a21=d/f

g=10;

h=500;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 40 e^(-0.080)t + 120 e^(-0.0160)t

% Y2 = -53.332 e^(-0.080)t + 480 e^(-0.0160)t

**8.3 Part 3 Matlab Code (Flow Changes)**

**8.3.1 Run 1**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 0.7159 e^(-0.0045)t + 159.284 e^(-0.1103)t

% Y2 = 0.0339 e^(-0.0045)t - 1680.03 e^(-0.1103)t

**8.3.2 Run 2**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=2;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 2;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 1.1831 e^(-0.0041)t + 158.81 e^(-0.1209)t

% Y2 = 0.0511 e^(-0.0041)t - 920 e^(-0.1209)t

**8.3.3 Run 3**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=5;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 5;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 1.8377 e^(-0.0033)t + 158.16 e^(-0.1517)t

% Y2 = 0.0626 e^(-0.0033)t - 464.06 e^(-0.1517)t

**8.3.4 Run 4**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=10;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 10;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 2.0244 e^(-0.0025)t + 157.97 e^(-0.2025)t

% Y2 = 0.0512 e^(-0.0025)t - 312.049 e^(-0.2025)t

**8.3.5 Run 5**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=100;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 100;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 0.6590 e^(-0.0005)t + 159.34 e^(-1.1045)t

% Y2 = 0.00296 e^(-0.-0005)t - 175.19 e^(-1.1045)t

**8.3.6 Run 6**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=2;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=2;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 158.44 e^(-0.1110)t + 1.5535 e^(-0.0090)t

% Y2 = -1600.08 e^(-0.1110)t + 0.1537 e^(-0.0090)t

**8.3.7 Run 7**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=2;

c=100;

a12=b/c

d=2;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=2;

f=200;

a21=d/f

g=10;

h=100;

i= 2;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 2.5203 e^(-0.0082)t + 157.47 e^(-0.1218)t

% Y2 = 0.2253 e^(-0.0082)t - 880.23 e^(-0.1218)t

**8.3.8 Run 8**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=5;

c=100;

a12=b/c

d=2;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=2;

f=200;

a21=d/f

g=10;

h=100;

i= 5;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 3.7937 e^(-0.0065)t + 156.20 e^(-0.1535)t

% Y2 = 0.2644 e^(-0.0065)t – 448.24 e^(-0.1535)t

**8.3.9 Run 9**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=10;

c=100;

a12=b/c

d=2;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=2;

f=200;

a21=d/f

g=10;

h=100;

i= 10;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 4.094 e^(-0.0049)t + 153.905 e^(-0.2051)t

% Y2 = 0.2094 e^(-0.0049)t - 300.29 80 e^(-0.2051)t

**8.3.10 Run 10**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=100;

c=100;

a12=b/c

d=2;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=2;

f=200;

a21=d/f

g=10;

h=100;

i= 100;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 1.3136 e^(-0.0009)t + 158.68 e^(-1.1091)t

% Y2 = 0.01195 e^(-0.0009)t - 174.4 e^(-1.1091)t

**8.3.11 Run 11**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=5;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=5;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 5.0208 e^(-0.0222)t + 154.97 e^(-0.1128)t

% Y2 = 1.4289 e^(-0.0222)t - 1361.43 e^(-0.1128)t

**8.3.12 Run 12**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=2;

c=100;

a12=b/c

d=5;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=5;

f=200;

a21=d/f

g=10;

h=100;

i= 2;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 7.619 e^(-0.02)t + 152.381 e^(-0.1250)t

% Y2 = 1.904 e^(-0.02)t - 761.9 e^(-0.1250)t

**8.3.13 Run 13**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=5;

c=100;

a12=b/c

d=5;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=5;

f=200;

a21=d/f

g=10;

h=100;

i= 5;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 10.368 e^(-0.0157)t + 149.631 e^(-0.1593)t

% Y2 = 1.9294 e^(-0.0157)t - 401.92 e^(-0.1593)t

**8.3.14 Run 14**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=10;

c=100;

a12=b/c

d=5;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=5;

f=200;

a21=d/f

g=10;

h=100;

i= 10;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 10.54 e^(-0.0117)t + 149.45 e^(-0.2133)t

% Y2 = 1.3937 e^(-0.0117)t - 281.38 e^(-0.2133)t

**8.3.15 Run 15**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=100;

c=100;

a12=b/c

d=5;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=5;

f=200;

a21=d/f

g=10;

h=100;

i= 100;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 3.2518 e^(-0.0022)t + 156.74 e^(-1.1228)t

% Y2 = 0.0741 e^(-0.0022)t – 172.07 e^(-1.1228)t

**8.3.16 Run 16**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=10;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=10;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 15.857 e^(-0.0126)t + 144.14 e^(-0.1174)t

% Y2 = 11.761 e^(-0.0126)t - 971.74 e^(-0.1174)t

**8.3.17 Run 17**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=2;

c=100;

a12=b/c

d=10;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=10;

f=200;

a21=d/f

g=10;

h=100;

i= 2;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 20.64 e^(-0.0328)t + 139.359 e^(-0.1322)t

% Y2 = 12.559 e^(-0.0328)t – 572.53 e^(-0.1322)t

**8.3.18 Run 18**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=5;

c=100;

a12=b/c

d=10;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=10;

f=200;

a21=d/f

g=10;

h=100;

i= 5;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 136.56 e^(-0.1707)t + 23.431 e^(-0.0293)t

% Y2 = -329.68 e^(-0.1707)t + 9.705 e^(-0.0293)t

**8.3.19 Run 19**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=10;

c=100;

a12=b/c

d=10;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=10;

f=200;

a21=d/f

g=10;

h=100;

i= 10;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 21.791 e^(-0.0219)t + 138.208 e^(-0.2281)t

% Y2 = 6.119 e^(-0.0219)t - 246.12 e^(-0.2281)t

**8.3.20 Run 20**   
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=100;

c=100;

a12=b/c

d=10;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=10;

f=200;

a21=d/f

g=10;

h=100;

i= 100;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 6.3979 e^(-0.0044)t + 153.6 e^(-1.1456)t

% Y2 = 0.2917 e^(-0.0044)t – 168.28 e^(-1.1456)t

**8.3.21 Run 21**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=100;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=100;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 4.7920 e^(-0.5124)t + 153.208 e^(-0.0976)t

% Y2 = -5.954 e^(-0.5124)t + 6165.47 e^(-0.0976)t

**8.3.22 Run 22**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=2;

c=100;

a12=b/c

d=100;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=100;

f=200;

a21=d/f

g=10;

h=100;

i= 2;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 9.2063 e^(-0.5247)t + 150.79 e^(-0.0953)t

% Y2 = -7.9708 e^(-0.5247)t + 3051.31 e^(-0.0953)t

**8.3.23 Run 23**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=5;

c=100;

a12=b/c

d=100;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=100;

f=200;

a21=d/f

g=10;

h=100;

i= 5;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 20.64 e^(-0.5608)t + 139.35 e^(-0.0892)t

% Y2 = -25.11 e^(-0.5608)t + 1145.03 e^(-0.0892)t

**8.3.24 Run 24**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=10;

c=100;

a12=b/c

d=100;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=100;

f=200;

a21=d/f

g=10;

h=100;

i= 10;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 35.43 e^(-0.6193)t + 124.56 e^(-0.0807)t

% Y2 = -42.25 e^(-0.6193)t + 522.23 e^(-0.0807)t

**8.3.25 Run 25**  
% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=100;

c=100;

a12=b/c

d=100;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=100;

f=200;

a21=d/f

g=10;

h=100;

i= 100;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 48.754 e^(-0.0319)t + 111.24 e^(-1.5681)t

% Y2 = 22.824 e^(-0.0319)t - 118.81 e^(-1.5681)t

**8.4 Part 1 Matlab Code (Including Plots)**

**8.4.1 Run 1**

clear

clear all

clc

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=100;

a12=b/c

d=16;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=200;

a21=d/f

g=10;

h=100;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

roots(P)

% Eigenvalues

Eigenvalues = eig(A)

% Eigenvectors

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1);

V2 = V(:,2);

V22 = V2/V(1,2);

Vectors = table(V22,V11)

% Displaying Eigenvalues and Eigenvectors in a table

K = table (Eigenvalues,V22,V11)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

% Solving for Constants

C = inv(J)\*b

% Y1 = 40 e^(-0.2)t + 120 e^(-0.04)t

% Y2 = -80 e^(-0.2)t + 80 e^(-0.04)t

% To plot y(t) we must model y(t) = C1(y1)+C2(y2)

% Where C are the constants C1 and C2

t = [0 27.5 50 100]';

y = [160 40.1080 16.2420 2.1979]';

T = (0:27.5:50:100)';

% Modeling y(t) = C1(y1)+C2(y2)= 40 e^(-0.2)t + 120 e^(-0.04)t

Y1 = [exp(-0.2\*T) exp(-0.04\*T)]\*C;

% % % Solving for T and Y

% Modeling y(t) = C1(y1)+C2(y2)= -80 e^(-0.2)t + 80 e^(-0.04)t

z =[-80 80]'

Y2 = [exp(-0.2\*T) exp(-0.04\*T)]\*z

% Displaying Tables for T,Y1 and T2,Y2

B = table(T,Y1)

w = table(T,Y2)

plot(T,Y1,'-',t,y,'o',T,Y2,'r')

title('y(t) vs. Time (Minutes)')

xlabel('Time (Minutes)')

ylabel(' y(t)')

grid

**8.5 Part 2 Matlab Code (Including Plots)**

**8.5.1 Run 1**

clear

clear all

clc

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=6;

c=20;

a12=b/c

d=16;

f=50;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=16;

f=50;

a21=d/f

g=10;

h=20;

i= 6;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 31.01 e^(-0.1681)t + 128.98 e^(-0.9519)t

% Y2 = 15.70 e^(-0.1681)t – 271.70 e^(-0.9519)t

% To plot y(t) we must model y(t) = C1(y1)+C2(y2)

% Where C are the constants C1 and C2

t = [0 27.5 50 100]';

y = [160 0.33142 0.0069384 1.5524e-06]';

T = (0:27.5:50:100)';

% Modeling y(t) = C1(y1)+C2(y2)= 31.01 e^(-0.1681)t + 128.98 e^(-0.9519)t

Y1 = [exp(-0.1681\*T) exp(-0.9519\*T)]\*C;

% % % Solving for T and Y

% Modeling y(t) = C1(y1)+C2(y2)= 15.70 e^(-0.1681)t – 271.70 e^(-0.9519)t

z =[15.70 -271.70]'

Y2 = [exp(-0.1681\*T) exp(-0.9519\*T)]\*z

% Displaying Tables

B = table(T,Y1)

w = table(T,Y2);

plot(T,Y1,'-',t,y,'o',T,Y2,'r')

title('y(t) vs. Time (Minutes)')

xlabel('Time (Minutes)')

ylabel(' y(t)')

grid

**8.6 Part 3 Matlab Code (Including Plots)**

**8.6.1 Run 1**

% Tank 1

% Y'= Inflow/Min - Outflow/Min

% Y'= -a11(Y1)+ a12(Y2)

b=1;

c=100;

a12=b/c

d=1;

f=200;

a11=-d/f

% Tank 2

% Y'= Inflow/Min - Outflow/Min

% Y'= a21(Y1)+ a22(Y2)

d=1;

f=200;

a21=d/f

g=10;

h=100;

i= 1;

a22= -i/h-g/h

A =[a11 a12;a21 a22]

P = poly(A)

Eig = eig(A)

[V,D] = eig(A);

V1 = V(:,1);

V11 = V1/V(1,1)

V2 = V(:,2);

V22 = V2/V(1,2)

% C1 + 1.5C2 = 160

% -2C1 + C2 = 0

J = [V22,V11]

b = [160;0];

C = inv(J)\*b

% Y1 = 0.7159 e^(-0.0045)t + 159.284 e^(-0.1103)t

% Y2 = 0.0339 e^(-0.0045)t - 1680.03 e^(-0.1103)t

% To plot y(t) we must model y(t) = C1(y1)+C2(y2)

% Where C are the constants C1 and C2

t = [0 27.5 50 100]';

y = [160 8.7399 1.2129 0.45906]';

T = (0:27.5:50:100)';

% Modeling y(t) = C1(y1)+C2(y2)= 40 e^(-0.2)t + 120 e^(-0.04)t

Y1 = [exp(-0.0045\*T) exp(-0.1103\*T)]\*C;

% % % Solving for T and Y

% Modeling y(t) = C1(y1)+C2(y2)= -80 e^(-0.2)t + 80 e^(-0.04)t

z =[0.0339 -1680.03]'

Y2 = [exp(-0.0045\*T) exp(-0.1103\*T)]\*z

% Displaying Tables

B = table(T,Y1)

w = table(T,Y2)

plot(T,Y1,'-',t,y,'o',T,Y2,'r')

xlabel('Time (Mins)')

ylabel(' y(t)')

grid